

Unit 8: Sequences and Series ~ Learning Guide

Name: _____

Instructions:

Using a pencil, complete the following questions as you work through the related lessons. Show ALL work as is explained in the lessons. Do your best and ask your instructor if you don't understand any questions!

Lesson One

Arithmetic Sequences

1. Write the next three terms in each arithmetic sequence.

a) 2, 4, 6, ...

b) $-7, -2, 3, \dots$

c) 17, 10, 3, ...

d) 5.1, 6.4, 7.7, ...

e) $\frac{1}{5}, \frac{7}{5}, \frac{13}{5}, \dots$

f) $10x, 8x - 1, 6x - 2, \dots$

2. Determine whether the following sequences are arithmetic. If yes, then determine their common difference.

a) 5, 8, 11, ...

b) $-3, -10, -17, \dots$

c) 1, $-3, 5, \dots$

d) $-4, -2.3, -0.6, \dots$

e) $2, -1.8, -5.6, \dots$

f) $3, \frac{11}{4}, \frac{5}{2}, \dots$

g) 1.7, 1.7, 1.7, ...

h) $2x, 6x, 10x, \dots$

i) $x+2, -5, -x-10, \dots$

3. Determine the value of x given the following information:

a) $a = 3$, $d = 4$, $t_7 = x$

b) $t_1 = -2$, $d = 2.3$, $t_9 = x$

c) $a = -3$, $d = -7$, $t_5 = x$

d) $a = 10.1$, $d = -3$, $t_x = -13.9$

e) $a = -5$, $d = 6$, $t_x = 115$

f) $t_1 = -7$, $d = -12$, $t_x = -127$

g) $a = s$, $d = 2$, $t_8 = x$

h) $a = b$, $d = 3n$, $t_{10} = x$

i) $a = 3$, $d = x$, $t_7 = 33$

j) $a = x$, $d = -11$, $t_{12} = -98$

k) $t_1 = -13$, $d = x$, $t_9 = -69$

l) $a = x$, $d = -3.25$, $t_{17} = -25$

m) $a = -19$, $d = x$, $t_{17} = 0.52$

n) $a = n$, $d = x$, $t_7 = -11n$

o) $a = 3m$, $d = x$, $t_{13} = -12.36m$

4. Fill in the missing terms in each of the following arithmetic sequences by finding the arithmetic means.

a) $6, _, 20$

b) $-5, _, -1$

c) $-6, _, -14$

d) $2, _, _, 11$

e) $\frac{3}{8}, _, \frac{9}{8}$

f) $x + 1, _, 5x + 9$

5. If an arithmetic sequence has a first term of 7 and $t_4 = 16$ then determine d .

6. If an arithmetic sequence has $t_3 = -6.5$ and $t_5 = -3.9$ then determine d .

- If an arithmetic sequence has $t_3 = 8$ and $t_6 = 29$, then determine the first term.
- If an arithmetic sequence has $t_4 = -11.98$ and $t_8 = 1.1$, then determine t_2 .
- If an arithmetic sequence has $t_5 = 2x + y$ and $t_7 = -8x + 7y$, then determine t_4 .
- There are originally 126 students at the school dance. Every car pulling into the parking lot from then on drops off 4 more students. How many students are at the dance after the 17th car drops off their students?

8. If an arithmetic sequence has $t_4 = -11.98$ and $t_8 = 1.1$, then determine t_2 .

9. If an arithmetic sequence has $t_5 = 2x + y$ and $t_7 = -8x + 7y$, then determine t_4 .

10. There are originally 126 students at the school dance. Every car pulling into the parking lot from then on drops off 4 more students. How many students are at the dance after the 17th car drops off their students?

Lesson TwoArithmetic Series

1. Determine the sum of each of the following arithmetic series.

a) $5 + 8 + 11 + 14 + 17 + 20$

b) $-3 - 10 - 17 - 24 - 31 - 38$

2. Determine the algebraically the sum of each of the following arithmetic series.

a) $3 + 5 + 7 + \dots + 23$

b) $5 + 1 - 3 - \dots - 35$

c) $-4 - 2.3 - 0.6 + \dots + 13$

d) $2 - 1.8 - 5.6 - \dots - 43.6$

e) $1.4 + 1.3 + 1.2 + \dots + 0.1$

f) $x + 6x + 11x + \dots + 21x$

3. Determine the value of x given the following information:

a) $a = 5$, $d = 3$, $S_{10} = x$

b) $a = -4$, $d = 1.3$, $S_8 = x$

c) $t_1 = -2$, $d = -6$, $S_7 = x$

d) $a = c$, $d = 3$, $S_8 = x$

e) $t_1 = b$, $d = 5b$, $S_5 = x$

f) $t_2 = 5$, $d = 2$, $S_7 = x$

g) $t_1 = x$, $d = -3$, $S_{12} = -138$

h) $a = x$, $d = 3$, $S_{13} = 78$

i) $a = 1.1$, $d = x$, $S_{17} = 73.1$

j) $a = m$, $d = x$, $S_5 = 5m - 30$

k) $a = 3m + 5$, $d = x$, $S_8 = 24m + 28b + 68$

l) $a = x$, $d = 3 - s$, $S_{10} = 95 - 25s$

- If an arithmetic series has $a = 13.7$ and $t_3 = 12.3$, then determine S_3 .
- If an arithmetic series has first term of $a = 4$ and $t_3 = 8$, then determine S_{10} .
- If an arithmetic series has $t_2 = -12.5$ and $t_4 = -10.7$, then determine S_{15} .
- Helmut's parents gave him \$1 on his first birthday (which went into his brand new piggy bank). On his second birthday Helmut received \$3 and on his third birthday \$5. His parents

5. If an arithmetic series has first term of $a = 4$ and $t_3 = 8$, then determine S_{10} .

6. If an arithmetic series has $t_2 = -12.5$ and $t_4 = -10.7$, then determine S_{15} .

7. Helmut's parents gave him \$1 on his first birthday (which went into his brand new piggy bank). On his second birthday Helmut received \$3 and on his third birthday \$5. His parents

continued to give him money following an arithmetic sequence. How much money would Helmut have **in total**, in his piggy bank, on his 13th birthday?

8. The side lengths of a triangle form an arithmetic sequence. The perimeter of the triangle is 33cm , and the longest side is 15cm . Determine the length of the other two sides.
9. Given the arithmetic series with $S_6 = -15$, $S_7 = -28$, and $d = -3$ find S_4 .

Lesson ThreeGeometric Sequences

1. Determine whether the following sequences are arithmetic, geometric, or neither. Either show algebraically, or explain in words why you made your decision.

a) $2, 4, 8, \dots$ _____

b) $3, 0, -3, \dots$ _____

c) $8, 4, 1, \dots$ _____

d) $1, -1, 1, \dots$ _____

e) $\frac{1}{3}, -1, 3, \dots$ _____

2. Determine algebraically the exact common ratio for the following geometric sequences.

a) $2, -8, 32, \dots$

b) $4, 4.8, 5.76, \dots$

c) $-81, 27, -9, \dots$

d) $64, 96, 144, \dots$

e) $2, -2\sqrt{2}, 4, \dots$

f) $4, 12\sqrt{3}, 108, \dots$

g) $m^2, m^5, m^8, \dots$

h) $3a^6, 27a^4, 243a^2, \dots$

i) $1, (2x + 1), (2x + 1)^2, \dots$

j) $-3, _, -48, \dots$

k) $2a^4, _, 4a^8, \dots$

l) $6, _, _, 48, \dots$

3. Determine the exact value of x given the following information.

a) $a = 2, r = 3, t_7 = x$

b) $a = 81, r = \frac{1}{3}, t_{10} = x$

c) $t_1 = -4, r = -2, t_8 = x$

d) $t_2 = 6, t_3 = 12, t_{10} = x$

e) $t_2 = -8, t_4 = -2, t_9 = x$

f) $t_2 = 8, t_5 = 27, t_{10} = x$

g) $a = 10, r = 2, t_x = 2560$

h) $a = -32, r = \frac{3}{2}, t_x = -243$

i) $a = -32, r = -0.72, t_x = 1.66395$

j) $t_1 = m^2, r = 2m, t_7 = x$

k) $a = \frac{m^2}{b}, r = \frac{b}{m}, t_{21} = x$

l) $t_3 = 8m^2, t_6 = \frac{1}{m}, t_{12} = x$

4. If the sum of the first two terms of a geometric sequence is 15, and the sum of the second and third terms is 60. Determine the first three terms of the sequence algebraically.

5. The first term of a geometric sequence is 24. The sum of the second and third terms is -6 . Determine the common ratio.
6. If the following three terms $x, x + 5, x + 15$ form a geometric sequence, find a value for x .
7. Find the 5th term in the geometric sequence $2x^8, \frac{6x^5}{y}, \frac{18x^2}{y^2}, \dots$

Lesson Four
Geometric Series

1. Determine the value of x given the following information.

a) $a = 2, r = 3, S_7 = x$

b) $t_1 = -3, r = -2, S_{10} = x$

c) $a = 64, r = \frac{3}{2}, S_8 = x$

d) $a = 16807, r = -\frac{2}{7}, S_6 = x$

e) $t_1 = 27, r = \frac{1}{3}, S_7 = x$

f) $a = 81, t_4 = 24, S_{10} = x$

g) $t_2 = -12, t_5 = \frac{3}{2}, S_6 = x$

h) $r = 3, S_8 = 6560, a = x$

i) $r = \frac{3}{2}, S_7 = 2059, t_1 = x$

j) $r = \frac{1}{3}, S_6 = -40.\bar{4}, t_4 = x$

k) $a = 2, t_3 = 18, S_x = 19682$

l) $t_2 = 6, t_5 = -48, S_x = 4095$

m) $t_2 = 500, t_3 = 400, S_x = 2600.712$

2. An oil well produces 70,000 barrels of oil on day one. Production is decreasing by 10% a day.
 - a) How many barrels of oil were produced on day 26?

 - b) How many barrels of oil are produced in total up to, and including, day 20?

3. A helium balloon rises 120m the first minute after it is released, each subsequent minute it rises 3% less than the previous minute. How high is the balloon 10mins after release?

4. If you invest \$50 at the start of every year at 4% compounded annually, what is the value of the annuity at the end of 20 years?

5. Elli gives her son Paul 1¢ on his first birthday, 2¢ on his second birthday, 4¢ on his third birthday...etc. (That is Elli doubles the amount she gives her son every birthday).
- How much money does she give Paul on his 10th birthday?
 - How much does she give Paul on his 30th birthday?
 - How much did Elli give Paul up to, and including, his 35th birthday?

Lesson Five

Infinite Geometric Series

1. Determine algebraically the exact value of x given the following information.

a) $a = 24, r = -\frac{1}{2}, S_{\infty} = x$

b) $a = 625, r = \frac{4}{5}, S_{\infty} = x$

c) $t_1 = 81, r = -\frac{1}{9}, S_{\infty} = x$

d) $a = 27, t_3 = 3, S_{\infty} = x$

e) $a = 96, t_2 = 144, S_\infty = x$

f) $a = 81, S_\infty = \frac{729}{8}, r = x$

g) $r = -\frac{2}{7}, S_\infty = \frac{117649}{9}, a = x$

h) $t_1 = 40960, S_\infty = \frac{163840}{3}, r = x$

i) $r = -\frac{1}{4}, S_\infty = \frac{40960}{5}, a = x$

j) $t_1 = 20480, S_2 = 25600, S_\infty = x$

k) $t_2 = -2560, t_5 = 320, S_\infty = x$

l) $a = 3840, S_2 = 1920, S_\infty = x$

m) $r = -\frac{2}{3}, S_2 = 27, S_\infty = x$

n) $r = \frac{1}{2}, S_3 = 140, S_\infty = x$

2. A ball is dropped from a height of $8m$ and rebounds to 80% of its previous height. What is the total distance the ball has traveled before coming to rest?

3. An oil well produces 30, 000 barrels of oil on day one. Production is decreasing by 8% a day.

- a) How many full barrels of oil were produced on day 14?

- b) How many full barrels of oil are produced in total up to and including day 17?

- c) How many full barrels of oil were produced in total before the well runs dry?

4. A helium balloon rises $70m$ the first minute after it is released, each subsequent minute it rises 12% less than the previous minute. How high does the balloon rise in total?

5. Determine the bounds on x , so that the following geometric series have a finite sum.
 - a) $3 + 3(x + 2) + 3(x + 2)^2 + \dots$
 - b) $5 + (2x - 3) + \frac{(2x - 3)^2}{5} + \dots$

Lesson Six

Sigma Notation

1. Find a) the number of terms, b) the first term, c) the common ratio, and d) the exact value of the sum for each of the following series.

a) $\sum_{k=1}^7 3(2)^{k-1}$

b) $\sum_{x=-2}^5 \frac{1}{4} \left(-\frac{1}{2}\right)^x$

c) $\sum_{k=2}^5 4(2)^{1-k}$

2. Write using sigma notation: $\frac{3}{16} + \frac{3}{8} + \frac{3}{4} \dots 1536$

3. Simplify: $\sum_{x=3}^5 \log 2^x$

ANSWERS*Lesson One Arithmetic Sequences*

1. a) 8, 10, 12 b) 8, 13, 18 c) -4, -11, -18 d) 9.0, 10.3, 11.6 e) $\frac{19}{5}, \frac{25}{5}, \frac{31}{5}$ f) $4x - 3, 2x - 4, -5$
 2. a) 3 b) -7 c) no d) 1.7 e) -3.8 f) $-\frac{1}{4}$ g) 0 h) $4x$ i) no
 3. a) 27 b) 16.4 c) -31 d) 9 e) 21 f) 11 g) $s + 14$ h) $b + 27n$ i) 5 j) 23 k) -7
 l) 27 m) 1.22 n) $-2n$ o) $-1.28m$
 4. a) 6, 13, 20 b) -5, -3, -1 c) -6, -10, -14 d) 2, 5, 8, 11 e) $\frac{3}{8}, \frac{3}{4}, \frac{9}{8}$ f) $x + 1, 3x + 5, 5x + 9$
 5. 3 6. 1.3 7. -6 8. -18.52 9. $7x - 2y$ 10. 194

Lesson Two Arithmetic Series

1. a) 75 b) -123
 2. a) 143 b) -165 c) 49.5 d) -270.4 e) 10.5 f) $55x$
 3. a) 185 b) 4.4 c) -140 d) $8s + 84$ e) $55b$ f) 63
 g) 5 h) -12 i) 0.4 j) -3 k) $b + 1$ l) $2s - 4$
 4. 39 5. 130 6. -106.5 7. \$169 8. $7cm$ and $11cm$ 9. 2

Lesson Three Geometric Sequences

1. a) Geometric b) Arithmetic c) Neither d) Geometric e) Geometric
 2. a) -4 b) 1.2 c) $-\frac{1}{3}$ d) $\frac{3}{2}$ e) $-\sqrt{2}$ f) $3\sqrt{3}$ g) m^3 h) $\frac{9}{a^2}$
 i) $2x + 1$ j) ± 4 k) $\pm a^2\sqrt{2}$ l) 2
 3. a) 1458 b) $\frac{1}{243}$ c) 512 d) 1536 e) $\pm \frac{1}{16}$ f) $\frac{6561}{32}$ g) 9 h) 6 i) 10 j) $64m^8$ k) $\frac{b^{19}}{m^{18}}$ l) $\frac{1}{64m^7}$
 4. 3, 12, 48 5. $-\frac{1}{2}$ 6. 5 7. $\frac{162}{x^4y^4}$

Lesson Four Geometric Series

1. a) 2186 b) 1023 c) 3152.5 d) 13065 e) $40.\overline{481}$ f) 238.786 g) 15.75
 h) 2 i) 64 j) -1 k) 9 l) 12 m) 8
 2. a) 5025 b) 614 896 3. $1050.3m$ 4. \$1548.46
 5. a) \$5.12 b) \$5 368 709.12 c) \$343 597 383.70

Lesson Five Infinite Geometric Series

1. a) 16 b) 3125 c) 72.9 d) 40.5 or 20.25 e) ∞ – no finite sum f) $\frac{1}{9}$
 g) 16807 h) $\frac{1}{4}$ i) 10240 j) $\frac{81920}{3}$ k) $\frac{10240}{3}$ l) 2560
 m) $\frac{243}{5}$ n) 160
 2. $72m$ 3. a) 10 147 b) 284 129 c) 375 000
 4. $583.\overline{3}m$ 5. a) $-3 < x < 1$ b) $-1 < x < 4$

Lesson Six Sigma Notation

$$1a) \text{ 7 terms } \quad a = 3 \quad r = 2 \quad S_7 = 381 \quad b) \text{ 8 terms } \quad a = 1 \quad r = -\frac{1}{2} \quad S_8 = \frac{85}{128} \quad c) \text{ 4 terms } \quad a = 2 \quad r = \frac{1}{2}$$

$$S_4 = \frac{15}{4} \quad 2 \sum_{k=1}^{14} \left(\frac{3}{16} \right) (2)^{k-1} \quad 3. \log 4096$$